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ARTIGO ORIGINAL

# Comparative study of the dimensioning of a floor of solid slabs using elastic, plastic, and numerical analysis

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**ABSTRACT:** A reinforced concrete structure design is carried out based on structural analysis results. From this, the engineer defines the most appropriate design technique for each type of building. Due to the importance of structural analysis, this paper proposed to carry out the analysis, dimensioning, and detailing of a floor of massive slabs, through three distinct methodologies, consisting of an elastic analysis, based on the Theory of Grids for deformable supports, a plastic analysis based on the Theory of Rupture Lines, and a numerical analysis by the Finite Element Method (FEM), carried out using the SAP2000 software. The accomplishment of this research had as its main objective to evaluate the steel consumption demanded by each analysis methodology used, and to establish a comparative study about the soliciting efforts determined by the three methods of structural analysis. Thus, when analyzing the results, there were large discrepancies between the bending moments presented in each analysis methodology. Also, regarding steel consumption, there were considerable savings for the plastic model compared to the elastic model, and taking the numerical model as a parameter, an 11.52% higher steel consumption was observed for the elastic calculation.

**Keywords:** analysis; massive slabs; methods; sizing; SAP2000.

# *Estudo comparativo do dimensionamento de um piso de lajes maciças utilizando análise elástica, plástica e numérica*

**RESUMO:** *A elaboração do projeto de uma estrutura de concreto armado é executada com base nos resultados de uma análise estrutural, sendo a partir desta que o engenheiro define a técnica de dimensionamento mais adequada para cada tipo de edificação. Devido à importância da análise estrutural, este artigo propôs-se a realizar a análise, o dimensionamento e o detalhamento de um pavimento de lajes maciças, por meio de três metodologias distintas, consistindo estas em uma análise elástica, fundamentada na Teoria de Grelhas* *para apoios deformáveis, uma análise plástica, baseada na Teoria das Linhas de Ruptura, e uma análise numérica pelo Método dos Elementos Finitos (MEF), realizada com o emprego do software SAP2000. A realização desta pesquisa teve como principal objetivo avaliar o consumo de aço demandado por cada metodologia de análise empregada e estabelecer estudo comparativo acerca dos esforços solicitantes determinados pelos três métodos de análise estrutural. Desse modo, ao analisar os resultados, verificou-se grandes discrepâncias entre os momentos fletores apresentados em cada metodologia de análise. Além disso, quanto ao consumo de aço, constatou-se uma considerável economia no modelo plástico em comparação ao modelo elástico e, tomando o modelo numérico como parâmetro, observou-se um consumo de aço 11,52% maior para o cálculo elástico.*

**Palavras-chave:** *análise; dimensionamento; lajes maciças; métodos; SAP2000.*

## 1 Introdução

The structural analysis constitutes the most relevant stage of a structure's design process, being that the engineer obtains the necessary results for the application of the most adequate design technique. However, depending on the boundary conditions and the type of request in which the structure is submitted, care must be taken when choosing the type of analysis to be employed.

The choice of an inappropriate analysis procedure can cause a wrong design, resulting in an overmatched structure. According to Gonzalez (1997), overmated parts offer risks to their use and should be avoided, in this way, armor is used in the usual slabs that normally do not present these disadvantages. The dangerousness of the super-woven parts comes from the fact that they are subject to a sudden rupture, without flow, in an exceptional loading situation in the Ultimate Limit State (ELU), in addition to being an uneconomical dimensioning.

An essential part of any multi-story building, the slabs are responsible for receiving and supporting the efforts arising from the use of loads and transmitting them to the beams and columns. According to Carvalho and Figueiredo Filho (2017) from the perspective of structural analysis, the slabs are concrete slabs with a flat surface, being mainly subject to normal actions to their medium plane, in which the dimension perpendicular to the surface, called thickness, is much smaller than the others.

There are several types of slab, such as solid, ribbed, smooth, and precast. The use of a given model is agreed to the best structural design employed in the function of what is defined in the architectural project. Parameters such as how to use the building and the environmental conditions are preponderant in this decision process.

The massive slabs are widely used in residential buildings, due to the small spans, which are commonly used in these. In Araújo (2014), solid slabs can be defined as plates of uniform thickness, supported along their contour, with the supports being beams or masonry. It is also possible to attribute the frequent use of massive slabs to the already well-consolidated mastery of its execution techniques, in addition to their efficiency in satisfactorily meeting various requirements for the use of the property, such as those inherent to the economic, aesthetic, comfort and safety.

In the academic environment, there is a vast bibliography, containing several methods for dimensioning the floors of solid slabs, and among these, the most widespread are the grid theory, the breaking line theory, the equivalent grid analogy, the finite differences, and the Finite Element Method (FEM). In this paper, some of these methodologies were used, as a basis for the establishment of a comparative study.

According to Araújo (2014), the grid theory consists of a simplified method in which an elastic-linear behavior of the slab material is admitted. When using this, the calculation of the requesting efforts depends on the predefined support conditions. According to Guessi (2017), in the calculation process through the theory of rupture lines, the dimensioning is carried out considering the balance of the slab in its imminent collapse moment, that is, in the ELU.

With the advent of computer technology, numerical methods have been increasingly used, due to their better performance in terms of agility in the execution and precision of results. According to Oñate (2009), the conceptual difference between analytical methods and numerical methods resides in that the former seeks, through mathematical expressions, the exact solution to a problem, however, these are only possible for some particular cases (which represent gross simplifications of reality), numerical methods aim to provide an approximate solution to the equations that govern the problem.

The advancement of information technology and the popularization of the computer enable the development of powerful and progressively more sophisticated algorithms. In this way, the production of commercial software makes use of numerical methods for the analysis of reinforced concrete structures, such as SAP2000, which uses the FEM to determine efforts and simulate the behavior of structures when subjected to a situation of loading.

With countless applications in the most varied areas of knowledge, FEM has been gaining more and more prominence in the analysis of reinforced concrete structures. According to Kattan (2008), the finite element method is a numerical procedure for solving engineering problems. The calculation process employing the FEM consists of the discretization of a domain in elements of reduced dimensions and defined geometry, linked together using nodes, in which the soliciting efforts are calculated based on their interactions.

An appropriate structural analysis allows the execution of a structure that meets all your requirements for use, and safety, and allows better use of the resources spent for its construction. Due to the availability of several structural analysis models, already disseminated in the literature, it is up to the responsible professional to list the methodology that best suits the conditions of its structure.

Each design methodology has particularities that directly influence execution costs. The expenditures spent on the various inputs inherent to the process, such as the consumption of steel in reinforcement, for example, are related to the magnitude of the efforts that apply to the structure, which come from structural and variable analyzes according to the calculation method employed. Thus, any possibility of savings combined with quality and safety becomes relevant in the current context of the crisis in which the competitive labor market in civil construction finds itself.

Because of this engineering assignment regarding the choice of the most appropriate model for structural analysis, this work proposed to promote a comparative study between some methodologies used for the design of a massive slab floor. The research compares the results obtained from the elastic analysis, based on the grid theory for flexible supports; plastic analysis, based on the theory of rupture lines; and those found in the linear numerical analysis by the FEM, performed with the aid of the SAP2000 software.

To compare these methodologies, the dimensioning and detailing of the pavement reinforcements were carried out for all the analysis methods used in this stud,y and based on these, the consumption of steel spent by the three processes was raised, to determine the energy savings. reinforcement from each method. In addition, this study promoted an assessment of the soliciting efforts in each calculation situation, to identify possible divergences between its results.

## 2 Theoretical framework

Reinforced concrete slabs, in general, can be analyzed as slabs, this is due to their geometric characteristics and how the loads that act on them affect them.

As structural elements of fundamental importance in buildings with multiple floors, the slabs are responsible for receiving, resisting, and transmitting to the beams all the loads resulting from their use, such as equipment, furniture, and people. In this way, various types of slabs are used in the usual constructions, in this work only the design methodologies are applied to the solid slabs, which the idealized as concrete slabs with constant thickness, totally or partially supported.

### 2.1 Elastic analysis

The elastic dimensioning of solid slabs consists of the application of the fundamentals of plate bending theory to determine the stresses that apply to them. According to the propositions of Kirchhoff-Love explored in Szilard (1974), it is possible to calculate the efforts, stresses, and deformations that act on the massive reinforced concrete slabs, admitting some simplifying hypotheses:

- The plate consists of linear, homogeneous, and isotropic elastic material;
- The plate is much less thick than the other two dimensions;
- The deflections and rotations of the deformed average surface are small concerning the thickness of the plate and the unit, respectively;
- Flat sections remain flat after deformations;
- Deflections of the plate are normal to the undisturbed plane and normal stresses to the medium surface are negligible.

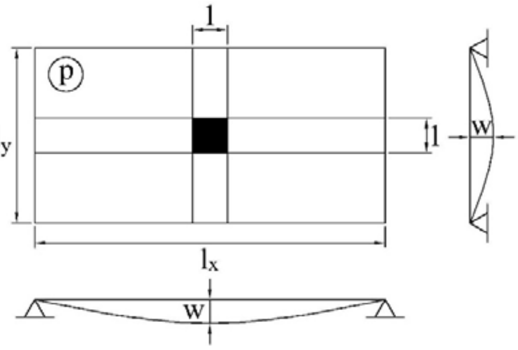
From the consideration of these hypotheses and the relationships between stresses and strains of elastic materials established by Hooke's law, the differential equation of the plate is obtained,

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p(x,y)}{D} \qquad (1)$$

where $w = w(x,y)$ it is the deformed or transverse displacement of the median plane of the plate, being $p(x,y)$ a normal load to it.

As discussed by Araújo (2014), the necessary considerations for the elastic calculation of solid slabs are presented, based on the theory of plate bending. First, the slab shown in Figure 1 is analyzed, in which two strips of unitary width are indicated in the directions $x$ and $y$ intersecting at its center, in which $p$ it corresponds to a load uniformly distributed per unit area. In this way, null arrows are obtained over it assuming that the supports are non-deformable in all their contour.

Figure 1 ▶
Slab supported on a rigid outline.
*Source: Araújo (2014)*



Observing the figure above and in front of what was exposed, it is noticed that the arrows in the directions $x$ and $y$ in the center of the slab are the same, but their curvatures are different because $l_x > l_y$. However, based on the plate bending theory, the bending moments $M_x$ and $M_y$ in the directions $x$ and $y$ are expressed through the following expressions:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right) \qquad (2)$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right) \qquad (3)$$

where

$$D = \frac{Eh^3}{12(1-v^2)} \qquad (4)$$

For the previous equations, $h$ the thickness and $D$ the flexural rigidity of the plate, and $E$ the longitudinal deformation modulus , and $v$ the Poisson's coefficient of the material, respectively. In addition, the tepartialrms $\frac{\partial^2 w}{\partial x^2}$ and $\frac{\partial^2 w}{\partial y^2}$express the curvatures according to the directions $x$ and $y$, respectively. It is agreed that for $l_x$ much greater than that $l_y$, the curvature is $\frac{\partial^2 w}{\partial x^2}$ negligible, from equations (2) and (3) it is obtained $M_x = vM_y$. As for concrete $v$ = 0,2, the last expression can be rewritten as follows $M_x = M_y/5$.

Based on the analyzes carried out by Araújo (2014), it is concluded that the bending moment is greater in the direction of the smaller span. From this observation, the slabs can be classified as cross-armed and reinforced in one direction. This distinction is due to the relationship between the gaps.

For slabs in which the ratio between the largest and the smallest span is less than 2, it is referred to as a cross reinforcement, in this case, the steel area of the reinforcements in both directions is calculated. As for the slabs in which the ratio between the largest and smallest span is greater than 2, the classification of the reinforced slab in one direction is attributed, in which the steel area is calculated only for the moment of greatest intensity in the direction of the smallest span and in the other direction a distribution armature is defined.

## 2.2 Plastic analysis

The plastic dimensioning of solid reinforced concrete slabs is carried out using the theory of rupture lines (TLB) or the theory of plastic hinges. As a result, their main underlying hypotheses are presented.

In the application of the TLB, the calculation of the soliciting efforts of the slabs to which the section of reinforced concrete resists, formerly called the last moments of plasticization, is carried out at the moment of imminent rupture, that is, in the ELU, these moments are obtained from a configuration plasticization lines or rupture lines.

According to Guessi (2017) for the application of the TLB, it is admitted that the slabs are subdivided into panels that, when they are under the action of rupture load, revolve around lines, in which the last moments of plasticization act, according to their normal direction. In this way, knowing the position of the rupture lines, and establishing the conditions of static equilibrium of contour in the panels delimited by them, it is possible to obtain the relationship between the moment of rupture and the ultimate load of the slab.

Gonzalez (1997) states in his work that the TLB consists of the application of the Kinematic Theorem or the Upper Limit of the plastic calculation that determines a load value greater than or equal to the ruin load, but that in general is lower than that found in the experimental results, due to the reserve of resistance resulting from the hardening of the steel and membrane effects of the slabs. Based on this idealization, some calculation hypotheses must be admitted:

- The material is considered rigid-plastic;
- The slabs must be sub-reinforced;
- Along and in the vicinity of each line of rupture, the bending moment is considered constant and equal to the maximum moment that the slab can resist;
- There is no premature rupture by shear or puncture;
- Resistance reserves are neglected.

The slabs must be dimensioned in their ruin configuration, this being the one where the highest value for the plasticization moment occurs. To determine the configuration of the rupture lines, some factors must be taken into account:

- Support conditions – Along with the extension of the crimped edges, lines of superior or negative ruptures are formed, corresponding to the negative moments;

- Nature and distribution of loads – Distributed loads result in straight rupture lines, while concentrated loads result in curved rupture lines;
- Armature arrangement – The slab service conditions are defined according to the adopted armature arrangement.

## 2.3 Numerical analysis

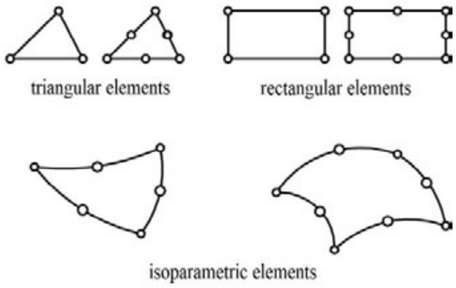
The last analysis addressed in the present work encompasses the procedures inherent to the numerical calculation of reinforced concrete massive slabs. The study consists of using the FEM, through the SAP2000 software, to perform the modeling and calculation of the stresses requesting slabs, in which the properties of the slabs executed in concrete are imposed.

The exact calculation of structures in general, even the simplest ones, requires the solution of very complex differential equations that in most cases cannot be obtained by analytical methods. According to Oñate (2009), to overcome this situation, gross simplifications of particular cases are used to solve structures with a low degree of complexity.

The FEM is an alternative for the more precise analysis of complex structures to present an approximate solution to the problem. According to Prazeres (2005), the method consists of subdividing a domain into smaller parts, called finite elements, which are composed of us. The analysis takes place through nodal interactions throughout the discretization mesh, determining the displacements, stresses, and deformations in the structure, for a given loading state and boundary conditions.

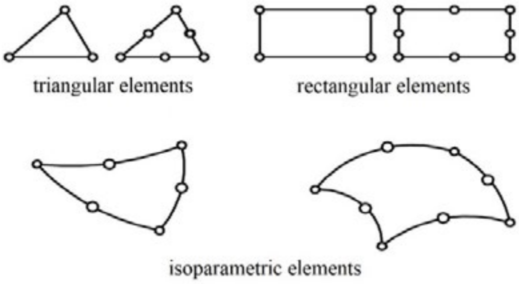
Souza (2003) reports that the FEM's main idea is to transform a complex problem into several simple solution problems, through an intuitive discretization process. In this way, continuous means are subdivided into finite element meshes with a defined geometry and limited dimensions, as shown in Figure 2.

Figure 2 ▶
Finite element mesh.
*Source: Souza (2003)*



According to Araújo (2014), the finite element is defined according to its number of nodes and geometry, with elements with regular shapes (triangular, rectangular, etc.) and isoparametric elements, which are distorted to better adapt the domain modeling irregular. Figure 3 shows some finite elements used for two-dimensional analysis.

Figure 3 ▶
Finite elements for two-dimensional analysis.
*Source: Araújo (2014)*



Each node of a finite element has degrees of freedom, translation, and rotation, which gives it possibilities for movement. Depending on the type of analysis you want to employ, restrictions on degrees of freedom are imposed. For the linear analysis of reticulated structures such as columns and beams, the use of unidimensional bar elements is common, whereas, for the discretization of plates, such as solid reinforced concrete slabs, two-dimensional rectangular elements can be used.

## 3 Research method

During the execution of this research, the structural form of the standard floor referring to the architectural project used in Fiorin (1998) was used for didactic purposes, to facilitate the reader's perception regarding the application of the inherent norms in the design of the structure. Figures 4 and 5 show, respectively, the floor plan and the structural form of the pavement, with all measures in meters.

Figure 4 ▶
A floor plan of the standard pavement.
*Source: Fiorin (1998)*

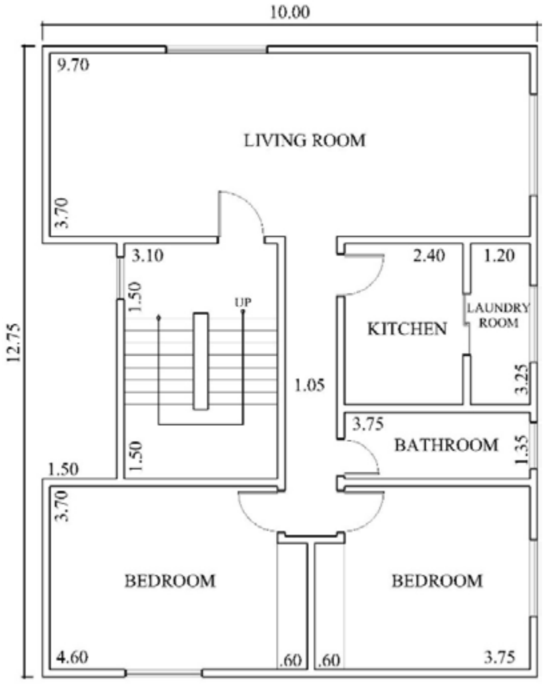
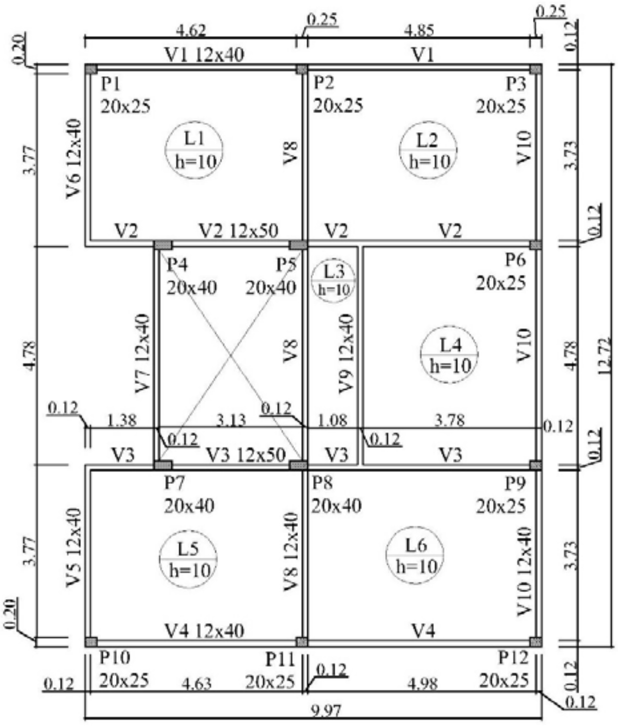


**Figure 5 ▶**
A floor plan of the standard pavement.
*Source: Fiorin (1998)*



To carry out the dimensioning of the pavement, a characteristic resistance to compression of 30 MPa concrete was adopted for all methodologies; longitudinal deformation module and drying module of concrete, calculated from the recommendations of NBR 6118 (ABNT, 2014), Poisson's ratio of 0.2; thickness for all 10 cm slabs; class of environmental aggressiveness II, for urban areas according to table 6.1 of NBR 6118 (ABNT, 2014), and also by the standard, nominal coverage for 25 mm slabs are used.

In Table 1, the permanent and variable loads that affect the slabs are indicated, which were obtained from Araújo (2014) and NBR 6120 (ABNT, 2019), in which the own weight is estimated by the expression 25*h*, with the thickness of the slab in meters. The L4 and L6 slabs have a localized overload from the walls that rest on them. In this work, we considered the weight of this masonry distributed over the entire area of the slab panel, the specific weight of which was obtained in the work of the aforementioned author.

**Table 1 ▶**
Loads acting on the slabs.
*Source: research data*

| Slab | Own weight (kN/m²) | Overload (kN/m²) | | |
|---|---|---|---|---|
| | | Coating | Accidental | Localized |
| L1 | 2.50 | 1.00 | 1.50 | 0.00 |
| L2 | 2.50 | 1.00 | 1.50 | 0.00 |
| L3 | 2.50 | 1.00 | 1.50 | 0.00 |
| L4 | 2.50 | 1.00 | 2.00 | 1.80 |
| L5 | 2.50 | 1.00 | 1.50 | 0.00 |
| L6 | 2.50 | 1.00 | 1.50 | 1.09 |

This research was carried out through the execution of a series of steps. Initially, the massive slabs of the floor were dimensioned using the theory of grids for deformable supports, using the expressions developed in Araújo (2014). For that, the slabs were classified according to their dimensions and support conditions, then the requesting efforts were determined from equations in line with the defined classification.

After the analysis based on elastic calculation, the second stage of the work was carried out, which deals with the plastic analysis for dimensioning the paving slabs, using the calculation routine developed in Gonzalez (1997) and based on the Theory of Lines Break (TLB). The procedures for the plastic calculation start with an elastic pre-dimensioning, using the approximate functions of Pinheiro (1988) based on the tables of Czerny (1976). The process continues with the determination of the plasticization moments, and then the calculation of the lengths for the negative reinforcement and verification of the arrows.

The next stage of the work consisted of performing a numerical analysis of the pavement using the FEM. The SAP2000 software is used for its execution to model the floor, using bar elements, called frames, for pillars and beams, and shell elements, called shells, for the slabs. The concrete class considered was C30. After the modeling of the structure was completed, the loading of the slabs was launched to simulate their behavior, with the requesting efforts for the design being determined from this.

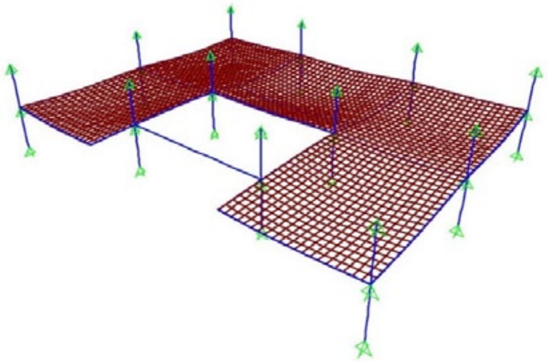
The last stage of this research comprised the realization of the dimensioning of the slabs from the efforts applied in the three analyzes. Then, the steel areas for the flexural reinforcements were determined and the admissible arrows were checked in each calculation hypothesis. The work continued by detailing the reinforcement and quantifying the consumption of steel spent for each analysis methodology.

Finally, a comparison was made between the efforts obtained in the three design cases, pointing out the particularities of each method, and how they influence the structure calculation process. A comparison of the consumption of steel required for each analysis was also established to promote the assessment of its impact on production costs. The cost of reinforcement varies according to the magnitude of the soliciting efforts, in addition, the increase or decrease of this type of input will directly impact the execution time and labor costs, increasing or reducing the execution costs of the project. structure.

## 4 Search results

The numerical analysis of the pavement was carried out with the help of the SAP2000 software, which makes use of the FEM to calculate the requesting efforts and to perform simulations of the structure's behavior when loaded. Thus, with the use of this program, pavement modeling was performed using a model elaborated according to the recommendations of NBR 6118 (ABNT, 2014) for linking between pillars and beams, which is illustrated in Figure 6.

Figure 6 ▶
Model for pavement analysis.
*Source: research data*



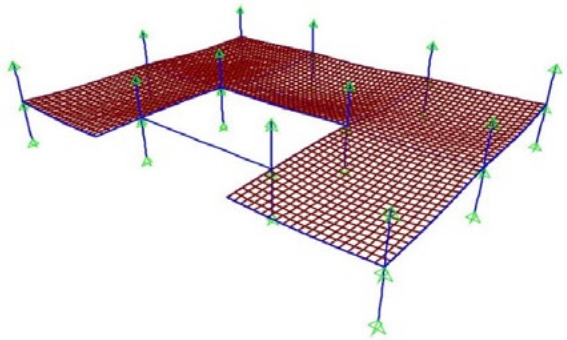
The model simulates for each column half the length of the upper and lower span. In its design, the beams and columns were modeled from frame elements, with all beams being attributed 15% of the torsional stiffness, according to the recommendations of item 14.6.6.2 of NBR 6118 (ABNT, 2014).

For carrying out the analysis, fixed supports were added at the ends of each column and in its links with the beams, thus, they were considered simply supported on the columns, as recommended by NBR 6118 (ABNT, 2014) in its item 14.6.6.1 for the study of vertical loads. In addition, no restriction was imposed on the vertical displacement of the beams, which are considered as flexible supports.

For the modeling of the slabs, the shell elements were used. The process took place as follows, initially, only one element was elaborated for each slab panel, which was supported in such a way that their average plane coincides with the axis of the beams, this procedure was adopted to simplify the model. Then, the elements were discretized, assigning a 20 × 20 mesh to each slab panel.

The modeling of the slabs continued with the launch of their respective permanent and variable loads. Figure 7 shows the deformed configuration of the pavement after carrying out the simulation.

Figure 7 ▶
Deformed configuration.
*Source: research data*



The soliciting efforts obtained through the simulation can be seen in Figures 8 and 9, with the first showing the active moments in the Direction $x$ and the second showing the active moments in the direction $y$. In addition, the maximum moments in each main direction were selected, which are shown in Table 2.

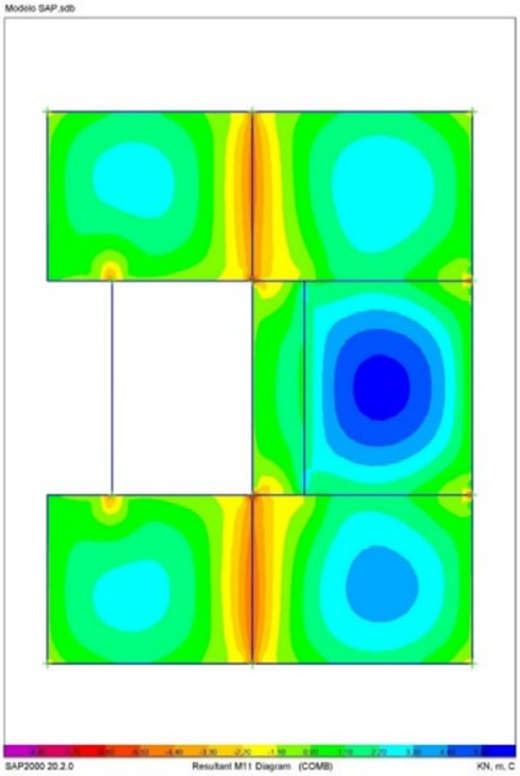


Figure 8 ▶
Bending moments in the direction *x*.
*Source: research data*

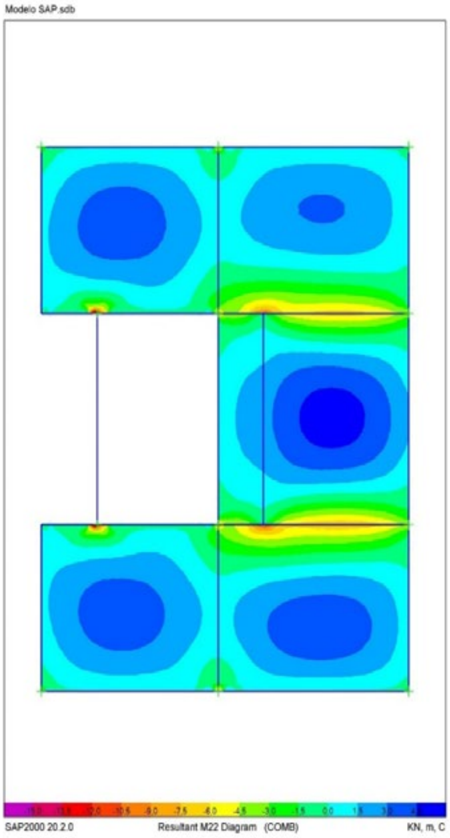


Figure 9 ▶
Bending moments in the direction of *y*.
*Source: research data*

Source: research data

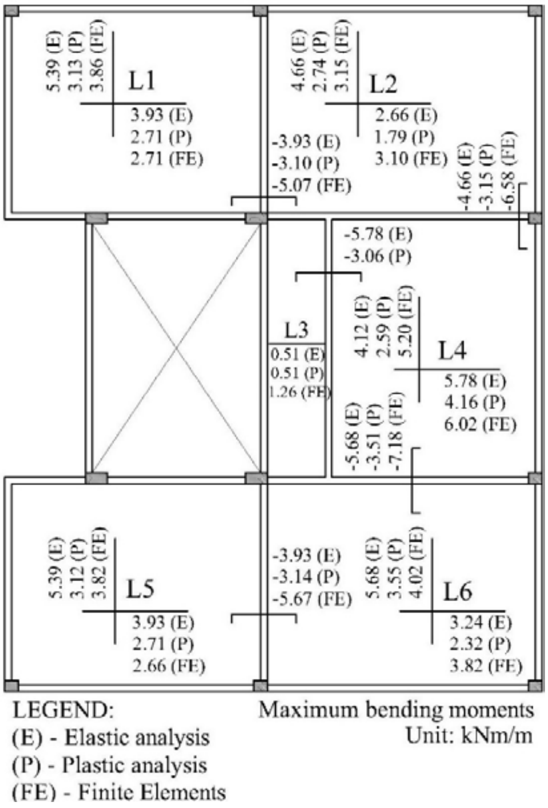
Table 2 ▶
Bending moments by FEM.
*Source: research data*

| Slabs | Positive moments | | Negative moments | |
|---|---|---|---|---|
| | Mx (kNm/m) | My (kNm/m) | Interface | Me (kNm/m) |
| L1 | 2.71 | 3.86 | L1-L2 | -5.07 |
| L2 | 3.10 | 3.15 | L2-L4 | -6.58 |
| L3 | 1.26 | 2.46 | L2-L3 | -5.41 |
| L4 | 6.02 | 5.20 | L3-L4 | - |
| L5 | 2.66 | 3.82 | L3-L6 | -5.50 |
| L6 | 3.82 | 4.02 | L4-L6 | -7.18 |
| - | - | - | L5-L6 | -5.67 |

The maximum bending moments were selected to avoid the peaks of effort concentration located at the support points, characterized by the pillars. As a reflection of these concentration peaks, negative bending moments have arisen in the regions comprising the L2-L3 and L3-L6 interfaces, which differs from the methodology, based on the elastic calculation, since the L3 slab is reinforced in one direction. As a result, negative reinforcement was calculated for these two regions.

To make the comparison between the results obtained through the elastic, plastic, and numerical analysis of the pavement slabs, Figure 10 was elaborated to promote a more adequate visualization of the data allowing a better confrontation of the same. This illustration shows the maximum negative and positive moments obtained in each analysis, which are arranged on the floor plan of the floor.

Figure 10 ▶
Maximum bending moments obtained from elastic, plastic, and numerical analysis.
*Source: research data*



In addition to Figure 10, Table 3 was created, which presents the positive moments obtained in each analysis, in which the data are compared based on the percentage difference of the values referring to the elastic and plastic analyzes compared to the moments found from the numerical analysis.

Table 3 ►
Comparison of positive moments.
*Source: research data*

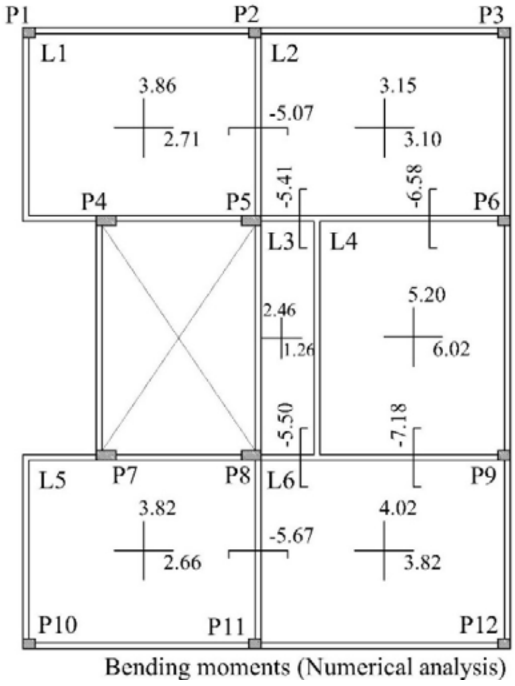
| Slab | Analyze | Mx (kNm/m) | DF% - FEM | My (kNm/m) | DF% - FEM |
|---|---|---|---|---|---|
| L1 | Elastic | 3.93 | 45.02 | 5.39 | 39.80 |
| | Plastic | 2.71 | 0.00 | 3.13 | -18.82 |
| | FEM | 2.71 | - | 3.86 | - |
| L2 | Elastic | 2.66 | -14.07 | 4.66 | 47.73 |
| | Plastic | 1.79 | -42.18 | 2.74 | -13.14 |
| | FEM | 3.10 | - | 3.15 | - |
| L3 | Elastic | 0.51 | -59.52 | - | - |
| | Plastic | 0.51 | -59.52 | - | - |
| | FEM | 1.26 | - | 2.46 | - |
| L4 | Elastic | 5.78 | -3.95 | 4.12 | -20.83 |
| | Plastic | 4.16 | -30.87 | 2.59 | -50.23 |
| | FEM | 6.02 | - | 5.20 | - |
| L5 | Elastic | 3.93 | 47.74 | 5.39 | 41.27 |
| | Plastic | 2.71 | 1.88 | 3.12 | -18.22 |
| | FEM | 2.66 | - | 3.82 | - |
| L6 | Elastic | 3.24 | -15.13 | 5.68 | 41.31 |
| | Plastic | 2.32 | -39.23 | 3.55 | -11.68 |
| | FEM | 3.82 | - | 4.02 | - |

Pinheiro (1988) states in his study that the cost of a slab is approximately proportional to the moments. In this way, it is interesting to evaluate these efforts. Given this observation and analyzing Figure 10, it can be seen that in general the positive moments obtained through the plastic analysis are considerably smaller than the moments found in the other analyzes. This fact can also be verified by Pinheiro (1988) and Gonzalez (1997).

Examining Table 3, it can be seen that when comparing the data, based on the results obtained from the FEM, a large percentage variation is observed in comparison with the values taken from the elastic and plastic analysis, and this oscillation is both positive in some cases. as negative in others, manifesting itself in a more expressive way for the elastic analysis, in which the discrepancy is greater than 40% in most confrontations.

In the case of the L3 slab, which is reinforced in one direction, there is greater divergence compared to other analyzes. As this panel was calculated in the same way for plastic and elastic analysis (due to the slab being unidirectional), the same value for the positive moment in the x-direction is justified. It can also be seen in Figure 11, in which the results of the numerical analysis are presented, an expressive value for the positive moment in the $y$-direction, however, this should not occur, as well as the absence of a negative moment at the L3-L4 interface. and its appearance at the L2-L3 and L3-L6 interfaces. As an explanation for these irregularities, one can attribute the influence of the concentration of efforts in the vicinity of the columns, in addition to the great curvature imposed by the bending of beam V9, which supports the slabs L3 and L4.

Figure 11 ▶
Maximum bending moments (numerical analysis).
*Source: research data*



Araújo (2008) explains that the traditional procedures for calculating reinforced slabs in one direction, based on plate theory do not present reliable values for slabs supported on flexible beams, and stresses that reinforced concrete slabs have a great capacity for redistributing efforts, which guarantees the safety of the traditional calculation.

In this work, the procedure proposed by Araújo (2014) was adopted, for the calculation of slabs supported on flexible beams, with divergent results of the analysis by the FEM, with the positive moment being higher, according to the direction of the largest span. However, it was found that this moment does not exceed the limit value for the minimum reinforcement requirement. Thus, for the dimensioning based on the numerical analysis, Araújo's (2008) guidance was followed, who recommends minimum reinforcement for the distribution of reinforced slabs in one direction.

For the analysis of the negative moments, Table 4 was elaborated, which also presents as a reference parameter in the calculation of the percentage variation, the moments obtained from the numerical analysis by the FEM.

Table 4 ▶
Comparison of negative moments.
*Source: research data*

| Interface | Analyze | Me (kNm/m) | DF% - FEM |
|---|---|---|---|
| L1-L2 | Elastic | 3.93 | -22.49 |
| | Plastic | 3.10 | -38.86 |
| | FEM | 5.07 | - |
| L2-L4 | Elastic | 4.66 | -29.23 |
| | Plastic | 3.15 | -52.16 |
| | FEM | 6.58 | - |
| L3-L4 | Elastic | 5.78 | - |
| | Plastic | 3.06 | - |
| | FEM | - | - |
| L4-L6 | Elastic | 5.68 | -20.87 |
| | Plastic | 3.51 | -51.10 |
| | FEM | 7.18 | - |
| L5-L6 | Elastic | 3.93 | -30.70 |
| | Plastic | 3.14 | -44.63 |
| | FEM | 5.67 | - |

Through an analysis of the data presented in Table 4, there is a large discrepancy in the negative moments in the plastic and elastic analyzes compared to the one carried out from the FEM, with the values found for the latter being considerably higher than the others. Such divergence is greater for the plastic analysis, being on average above 45%, although smaller, the difference is still quite accentuated for the elastic analysis being on average above 20%.

The reduction in the negative moments of the elastic analysis, in comparison to the analysis by the FEM, can be attributed to the fact that the theory of grids for deformable supports was used in its realization, which roughly considers a redistribution of efforts from the supports, thus reducing negative moments. It should also be taken into account that the numerical analysis in this work did not include the redistribution of efforts.

As for the plastic analysis, the large discrepancy of the negative moments to the values found by the FEM can be justified by the elastic pre-dimensioning, proposed in Pinheiro (1988) and Gonzalez (1997), which precedes the plastic calculation, in which the moment's negatives are halved.

It is important to emphasize the finding, through a bibliographic review, that the discrepancies in the soliciting efforts found from the three analyzes are by the proportions contained in other academic works, as also stated by Fava and Neves (2018), in their work that evaluated the soliciting efforts for solid slab floors obtained from the analysis by the FEM and grid theory.

Furthermore, Werner and Vargas (2013) compared the Marcus method, the resolution by series, and the FEM for the dimensioning of bidirectional solid slabs was carried out, and it was found that the results demonstrate differences that do not present a defined pattern, the variations found being justified by the different concepts on which each structural analysis methodology is based.

It is still pertinent to mention that the verification of the arrows was satisfied for all the design methodologies, however, in the numerical analysis, the values found are much lower than those presented in the other analyzes. However, a more in-depth assessment of the study of arrows will not be carried out, as this is not the main focus of the scope of this research.

Table 5 indicates a comparison of the steel consumption demanded by each dimensioning methodology. In it are tabulated the results obtained for the weight of steel in each method (both for positive and negative reinforcements) and the percentage differences between elastic, plastic, and numerical analyzes.

Table 5 ▶
Comparison of steel consumption.
*Source: research data*

| **Armor** | **Analyze** | **Steel consumption (Kg)** | **DF (%) - FEM** | **DF (%) - Elastic** | **DF (%) - Plastic** |
|---|---|---|---|---|---|
| Positive | Elastic | 382.28 | 21.79 | - | 25.74 |
| | Plastic | 304.03 | -3.14 | -20.47 | - |
| | FEM | 313.89 | - | -17.89 | 3.24 |
| Negative | Elastic | 85.14 | -19.11 | - | -22.25 |
| | Plastic | 109.50 | 4.03 | 28.61 | - |
| | FEM | 105.26 | - | 23.62 | -3.88 |
| Total | Elastic | 467.42 | 11.52 | - | 13.03 |
| | Plastic | 413.53 | -1.34 | -11.53 | - |
| | FEM | 419.14 | - | -10.33 | 1.36 |

Looking at Table 5, it can be seen that plastic analysis is the methodology that presents the least consumption of steel, with savings of 13.03% to the elastic calculation and 1.36% in comparison with the analysis by the FEM. However, the plastic procedure is the one that presents the highest consumption of steel for the negative reinforcement, this fact is due to the high length calculated for these fittings, that despite the small magnitude of the negative moments, it is necessary to have longer lengths for these rebars so that the TLB sizing assumptions are met.

In contrast to plastic sizing, elastic analysis is the one with the highest consumption of steel, 11.52% higher than that found by FEM. However, it is also the methodology with the lowest demand for negative reinforcement, and consequently the one with the highest consumption of steel for positive reinforcement. This fact can be justified by the consideration of deformable supports using the grid theory for elastic calculation, in which, as previously discussed, negative moments are reduced, to simulate a redistribution of efforts.

It is still relevant to mention, that the three methodologies meet the restrictions imposed by the limitation of the allowable arrows in the Excessive Deformation Service Limit State (ELS-DEF), even obtaining smaller arrows for the numerical analysis, when compared to the others. In this way, it can be said that all methodologies could be used for dimensioning the pavement, providing a reasonable safety margin.

## 5 Conclusion

In this paper, a comparative study was developed about three methodologies for the analysis, dimensioning, and detailing of a massive slab floor. These were elastic, plastic, and numerical analysis. In the elastic calculation, the grid theory for deformable supports was used, since the plastic dimensioning was done through the application of the TLB, and finally, the numerical analysis was elaborated from the FEM using the SAP2000 software.

From the results obtained through the execution of each dimensioning methodology, a confrontation of data was carried out about the positive and negative moments found in the three analyzes, in which a large discrepancy of the results was found when taking the FEM as a reference, especially among positive moments. The plastic calculation was the one that generally presented the lowest values, both for positive and negative moments.

The study carried out in this research also extended to the comparison between the sizing methods about the consumption of steel demanded each analysis employed, with greater savings in reinforcement for the plastic calculation, which showed results very close to consumption. determined by the FEM. On the other hand, the elastic dimensioning was the one that showed the highest demand for reinforcement, being 11.52% higher, when compared to the FEM.

In short, it can be concluded that the plastic dimensioning presents considerable savings regarding the consumption of steel concerning the other methods, when applied to the floor used in this research, which is quite representative of the usual residential buildings. In addition, the elastic calculation using the grid theory for deformable supports, in some cases, when compared to the FEM, can result in over-dimensioning, based on the highest consumption of steel found. Thus, a more in-depth study of the effects of the redistribution of efforts used to consider flexible support is necessary.

In addition, it is relevant to emphasize that all the design methodologies performed in this work, met the checks imposed by the limitations of the arrows. Thus, even if a deeper study on the subject has not been carried out, it can be said that the analyzes presented here have an adequate safety margin, for their application in the design of massive slabs.

In the possibility of future work, in which the continuity of this research is carried out, it is interesting to carry out a numerical analysis in which the redistribution of efforts is employed, to reduce the effects caused by the peak moments found in the support regions, to that we can draw a comparison with the elastic calculation used in this work, in which the flexibility of the supports was considered. In addition, a more detailed study can be carried out on the issue of verifying arrows, mainly for numerical analysis.

A better understanding of the topic of numerical analysis of reinforced concrete pavements with the FEM can be important for the refinement and understanding of the model (even for the linear elastic case in question).

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