

# A Convex Formulation for Voltage Unbalance Compensation Problem on Hybrid Microgrids

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## ABSTRACT

Voltage unbalance is a power quality condition that arises due to the presence of unbalanced single-phase loads. The presence of voltage unbalances affects negatively a series of sensible equipment and, therefore, is undesirable. In a hybrid (AC/DC) microgrid environment, the presence of distributed generators connected by converters to the AC side can be used to tackle the voltage unbalance. This work presents a convex optimal voltage unbalance compensator that uses the converters' capability to adjust the negative sequence voltage of their buses in order to keep the overall voltage unbalance of the network within a given range. The voltage unbalances compensation effort sharing between the converters is formulated as a quadratic constrained quadratic programming (QCQP) problem whose convexity assures the global optimality of the solution. The formulation is based on the equivalent negative sequence circuit of the network. The technique was evaluated by simulation on a study case microgrid and was able to successfully reduce the voltage unbalance to desirable levels.

**Keywords:** Voltage Unbalance Compensation. Hybrid Microgrid. Quadratic Constrained Quadratic Programming.

## RESUMO

*O desbalanço de tensão é um problema de qualidade de energia que surge na presença de cargas monofásicas desbalanceadas. A presença de desbalanço de tensão afeta negativamente uma série de equipamentos sensíveis e, portanto, é indesejável. Em uma microrrede híbrida (CA/CC), a presença de geradores distribuídos conectados ao barramento AC por conversores pode ser usada para amenizar o desbalanço de tensão. Este trabalho apresenta um compensador de desbalanço ótimo que faz uso da capacidade dos conversores de ajustar a tensão de sequência negativa dos respectivos barramentos, objetivando manter o desbalanço de tensão dentro de níveis aceitáveis. O problema de compartilhamento do esforço de compensação de desbalanço entre os conversores é formulado como um problema de programação quadrática com restrições quadráticas (QCQP, Quadratic Constrained Quadratic Programming), cuja convexidade assegura a otimalidade global da solução. A formulação é baseada no circuito equivalente de sequência negativa da rede. A técnica proposta foi avaliada por simulação em uma microrrede de estudo de caso e foi capaz de reduzir o desbalanço de tensão para níveis desejáveis.*

**Palavras-Chave:** *Compensação de Desbalanço de Tensão. Microrrede Híbrida. Programação Quadrática Restrita Quadrática.*

# 1 Introduction

Non-renewable resources, such as diesel, coal, and gas, have been playing a major role in the traditional power generation. However, with a 2.5 % annual load growth, an unmatched gap is arising in between demand and conventional power generation (SEN; KUMAR, 2018). Along with depletion of reserves of non-renewable resources, the environmental pollution resulting of the large usage of non-renewable resources have caused the power generation scenario to start shifting to more environmentally friendly energy resources (FAN et al, 2012).-

The development of cost-effective sources tailored to provide generation in smaller quantities, such as photovoltaic panels and microturbines, allowed the dissemination of Distributed Generation (DG), creating a new paradigm where the generation units are located near the consumers, offering support to the main sources (OLIVARES et al, 2014).

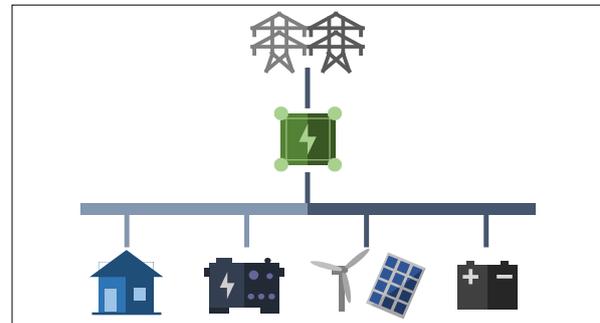
Parallel to this, the advances in information and communications technologies have paved the road to smart grids: more reliable and flexible networks, integrating distributed generation and smart houses (FAN et al, 2012).

Regarded as the elementary units of a smart grid, microgrids (MGs) have undergone vigorous research for more than one and a half decade and are a technology brewed to face those ever-growing challenges. The MG paradigm is not only economical, resilient and reliable but also provides environmental benefits as compared to the existing utility networks because of the use of Renewable Energy Resources (RESs) in a distributed generation fashion. Microgrids can be classified as DC, AC or hybrid (AC/DC), regarding the used transmission method between each bus of the microgrid (SEN; KUMAR, 2018).

The MG operation is coordinated by a Microgrid Central Controller (MGCC), which takes care of the power flow between the main grid and the MG, optimizing the MG operational cost, deciding the operation mode, controlling the generation dispatch and energy storage management (LI; NEJABATKHAH, 2014). Figure 1 illustrates the concept of a MG, where loads, energy storages, renewable and non-renewable DGs are connected to a MGCC which also manages the connection to the main grid. In particular, on hybrid MGs, there are at least a DC and an AC bus. Energy storages and renewable generation are usually connected to the DC buses, while loads and

non-renewable generators are usually connected to AC buses. DC/AC converters are employed to interface DC and AC buses.

**Figure 1** – A microgrid gathers loads, conventional and renewable generation, energy storage and deals with the main grid.

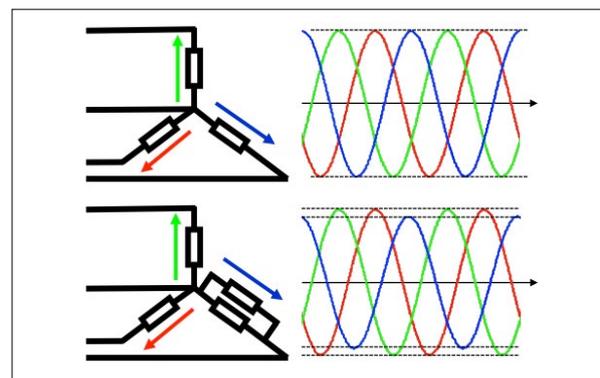


Source: author.

One aspect of the MG operation that must be tackled by MGCC is the presence of voltages unbalances on AC and hybrid (AC/DC) MGs. Voltage unbalances arise in three-phase systems mostly due to the presence of single-phase loads connected between one phase and the neutral or between two phases. This results in uneven load distribution over the phases. This is illustrated in Figure 2.

Even with careful design aiming to balance the loads evenly, a proper balance of single-phase loads among the three phases is difficult to keep in time-varying load operation condition (KIM et al, 2005). Additionally, asymmetrical line and transformer impedances, open wye and delta transformer banks and non-linear loads may also cause voltage unbalances (BLACK et al, 2015).

**Figure 2** – Balanced voltage on a symmetric load condition (above) and unbalanced voltages due to asymmetric load (below).



Source: author.

The presence of voltage unbalance degrades the performance and lifespan of sensible loads, such as three-phase induction motors, which experiment de-rating and overheating due to loss currents caused by voltage unbalances (KIM et al, 2005). Power electronics converters and adjustable speed drives also suffer from the presence of voltage unbalance (ACHARYA et al, 2019).-

In order to tackle the voltage unbalance several strategies have been proposed in the literature. The ideal approach would be simply redistributing the loads online. Chen and Cherng (2000) proposed a realization of this by rearranging and balancing phases in the primary side of the distribution transformer. Shahnia, Wolfs and Ghosh (2014) proposed switching loads among phases using static transfer switches on the residential environment instead.

A set of less explicit approximations of the ideal solution rely on demand-side response techniques, using, for example, electric vehicles (MARTINENAS; KNEZOVIC; MARINELLI, 2017) or thermostatically controlled loads (ACHARYA et al, 2019), even though demand-side based strategies may impact on user comfort.

Departing from the ideal solution and delving into active filters, both series and parallel active filters have been used operating in series with the network, injecting negative sequence voltage, or operating in parallel with the network, injecting negative sequence current (MENG et al, 2014).-

In a microgrid environment, the presence of DG can be exploited to address voltage unbalance issues. In that case, each DG unbalances its own output in order to mitigate the unbalance on the loads. The multiple DGs present on the system can share the compensation effort aiming to achieve the reduction of voltage unbalance under various strategies, several of them including a centralized control on a secondary level (MENG et al, 2017).-

An early proposal, from 2012, developed a two-level hierarchical control approach for voltage unbalance compensation where the MGCC receives measurements of positive and negative sequence voltages and uses a PI controller to calculate the correction negative sequence voltage set-point for each converter (SAVAGHEBI et al, 2011). A more elaborate scheme was presented by Meng et al. (2014), proposing a tertiary control strategy for unbalance compensation. Based on the studied case of Savaghebi et al., (2011), the authors proposed a new topology

adding a third level which employs a Genetic Algorithm (GA) to calculate optimal weights for the set-points found by the secondary level, in order to improve the division of correction effort between the multiple DGs. The proposed cost function describes the voltage unbalance excesses on each load bus. The weights are accounted as decision variables of the problem solved by the GA.

A similar approach was employed by Meng et al. (2017), where the authors adopt a two-level hierarchical control focusing on customized energy quality criteria when each load bus has a specific acceptable voltage unbalance level. The secondary level uses GA to minimize the voltage unbalance excesses on each load bus, using the negative sequence voltage injected by each DG as decision variables. The optimization problem uses a negative sequence equivalent circuit. The obtained values are sent to the local controllers.

Karagiannopoulos, Aristidou and Hug (2018) proposed a centralized operation scheme based on a multi-period optimal power flow algorithm used to compute optimal set-points of the controllable distributed energy resources located in the system. The proposed algorithm runs a three-phase multi-period Optimal Power Flow (OPF) after one Backward-Forward Sweep (BSF) iteration, to obtain the values for each dispatchable element and then runs the BSF until convergence. The process is repeated until the voltage mismatch is reduced to a certain level. The adopted strategy reduces the operational cost while satisfying the appropriate security and power quality constraints, in special, treating voltage unbalance within the OPF assuming that the positive sequence value is near unity (expressed in per-unit).

After reviewing the literature, a gap was noticed as the found solutions for the problem of the voltage unbalance compensation effort sharing between multiple converters had not been taking advantage of the optimality guaranteed by a convex formulation. Savaghebi et al. (2011) do not adopt an optimal approach. Meng et al. (2014) and Meng et al. (2017) improve the result of the first work but employ GA to do so, providing no guarantee of global optimality. Karagiannopoulos, Aristidou and Hug (2018), on the other hand, resort to simplification to avoid the division of two decision variable, thus keeping the simplified problem of the convex OPF.

Aiming to fill the gap, the present work revisits the approach presented in Meng et al. (2017), reformulating it as a quadratic constrained quadratic

programming problem (QCQP) to compose a voltage unbalance compensator (VUC) deployed on MGCC level to optimally share the voltage unbalance compensation effort among the converters on the MG. The proposed approach assumes the existence of phasor measurement of voltages and currents on each bus.

In the rest of this paper, the second section covers some basic aspects on voltage unbalance, the third section presents the formulation of the proposed strategy, the fourth section examines the study case microgrid used to assess the proposed approach, the fifth section comprises the simulation results and the sixth section brings some conclusions.

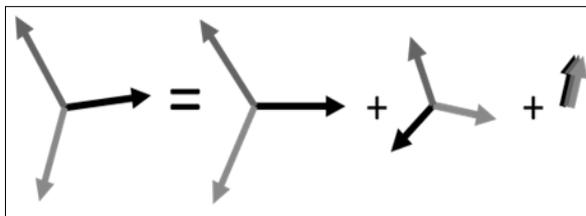
## 2 Voltage Unbalance Fundamentals

Given a three-phase unbalanced voltage signal, with phase voltages  $V_a, V_b, V_c$ :

$$V_{abc} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}, \quad (1)$$

the Fortescue theorem states that this unbalanced signal can be represented as a summation of three balanced signals (BLACKBURN, 1993), as illustrated by Figure 3.

**Figure 3** – An asymmetrical three-phase system is represented as the summation of three symmetrical systems. From left to right: a positive, a negative and a zero sequence system.



Source: author.

In order to do so, the Fortescue transform can be used to represent the unbalanced voltage as a sum of three balanced voltages of zero, positive and negative sequences:

$$V_{012} = \begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = A^{-1}V_{abc}, \quad (2)$$

where the inverse Fortescue transform matrix is given as:

$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}, \quad (3)$$

with  $\alpha = 1 \angle 120^\circ$  is the displacement operator, and  $V_0, V_1, V_2$  are the zero, positive and negative sequence components of the symmetrical components' representation. The positive sequence phasors rotate counter-clockwise (abc), while the negative sequence components rotate clockwise (acb) and the zero sequence components are in phase.

On a balanced system, the negative and zero components have null values and the positive sequence components equal to the original signal. The zero sequence components assume non zero values when there is a neutral voltage drift, which occurs on unbalanced systems, due to neutral current circulation, as the zero sequence components represent a displacement of the origin of the abc- frame. Likewise, the negative sequence components will assume non zero values in unbalanced systems. This fact is used in order to leverage a metric to quantify the unbalance, the Voltage Unbalance Factor (VUF), which is given as (KIM et al., 2005):

$$VUF = \frac{|V_2|}{|V_1|} \cdot 100\%. \quad (4)$$

Concerning the presence of voltage unbalance, there are several standards prescribing acceptable VUF values around the world. The most rigorous limit is found in the United Kingdom, which prescribes the maximum acceptable VUF at 1.3 %. A maximum VUF of 2.0 % is adopted in France, Germany, and European Union, while IEEE adopts 2.0 % to 2.5 % as an adequate limit (RUIZ-RODRIGUEZ; HERNÁNDEZ; JURADO, 2015). The International Electrotechnical Commission (IEC) and the International Council on Large Electric Systems (CIGRE) Working Group 36.07 also suggest a maximum VUF of 2% (ACHARYA et al., 2019). The 8th module of PRODIST, Brazilian Standards on Power Distribution (ANEEL, 2018), prescribes a maximum VUF of 3 % for grids operating under 1 kV.

## 3 Voltage Unbalance Compensator Formulation

Whenever an unbalanced load is connected to the system, that load bus will consume negative sequence current, which will flow through the line

negative sequence admittance creating a distortion in the phase voltages of the load bus, thus causing the appearance of negative sequence voltages, while the source bus voltages remain balanced. Provided that a DG converter on the source bus has the capability of controlling the negative sequence voltage in its own bus, it can adjust the negative sequence voltage of the source bus in order to reduce the negative sequence voltage on the load bus, thus distorting its own voltage aiming to achieve lower VUF values on the load side (Meng et al., 2017).

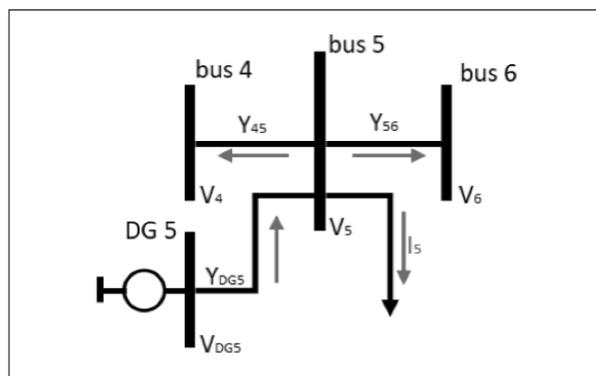
The existence of DGs on a MG can be used to tackle the voltage unbalance, but the nature of their distribution on the network requires the coordination of the MGCC to set the contribution each DG should provide so to improve the overall condition. That said, this problem is formulated as an optimization problem in this section.

Aiming to model the voltage unbalance compensation effort sharing problem as an optimization problem it is necessary to leverage a set of constraints that describe the steady-state electrical behavior of the microgrid with the negative sequence voltages on the DG buses as decision variables and a cost function that enable the problem to assess the VUF on each load bus.

Considering the  $j$ -th bus of the MG, which has a DG, a load and also being connected to  $M$  other buses. Figure 4 exemplifies such bus. Considering that the current from the DG enters the node and all of the other currents to the loads and the buses leave the node, it is possible to perform a nodal analysis for that bus on the negative sequence equivalent circuit:

$$(V_2^{jDG} - V_2^j)Y_2^{jDG} = I_2^j + \sum_m^M (V_2^j - V_2^m)Y_2^{jm} \quad (5)$$

Figure 4 – Nodal analysis for a bus on the MG.



Source: author.

Where  $V_2^{jDG}$  is the negative sequence voltage of the DG,  $V_2^j$  is the negative sequence voltage of the  $j$ -th bus,  $Y_2^{jDG}$  is the negative sequence admittance of the line between the DG and the  $j$ -th bus,  $I_2^j$  is the negative sequence current demanded by the load connected to the  $j$ -th bus,  $V_2^m$  is the negative sequence voltage of the  $m$ -th bus connected to the  $j$ -th bus and  $Y_2^{jm}$  is the negative sequence admittance of the line between the  $j$ -th bus and the  $m$ -th bus connected to it. If the  $j$ -th bus does not have a DG connected to it, the left-hand side of Equation (5) is zero. Conversely, if the bus has no load connected to it, the load current term is zero.

For a microgrid with  $N$  buses, a set of  $N$  nodal equations are included into the constraints in order to describe the negative sequence equivalent circuit.

Besides the inclusion of the model, it is also necessary to contemplate the current limitations of the DG in the constraints. For each DG it is imposed that:

$$|I_2^{jDG}| \leq |I_{2max}^{jDG}|, \quad (7)$$

with the negative sequence current supplied by the DG,  $I_2^{jDG}$  being computed as:

$$I_2^{jDG} = (V_2^{jDG} - V_2^j)Y_2^{jDG}, \quad (8)$$

and the maximum negative sequence current,  $I_{2max}^{jDG}$ , being obtained from the rated current limit of the DG and the measured positive and zero current sequence, as it must hold that:

$$|AI_{012}^{jDG}| \leq I_{\phi max}^{jDG} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad (9)$$

i.e.:

$$\begin{cases} |I_0^{jDG} + I_1^{jDG} + I_2^{jDG}| \leq I_{\phi max}^{jDG} \\ |I_0^{jDG} + \alpha^2 I_1^{jDG} + \alpha I_2^{jDG}| \leq I_{\phi max}^{jDG} \\ |I_0^{jDG} + \alpha I_1^{jDG} + \alpha^2 I_2^{jDG}| \leq I_{\phi max}^{jDG} \end{cases} \quad (10)$$

where  $I_{\phi max}^{jDG}$  is the maximum rated phase current of the DG,  $I_0^{jDG}$ ,  $I_1^{jDG}$  and  $I_2^{jDG}$  are respectively the zero, positive and negative sequence components of the  $j$ -th DG current.

The cost function that accounts for the excess VUF is:

$$J = \sum_j^N \max(VUF^j - VUF_{max}, 0), \quad (11)$$

where  $VUF_{max}$  is the maximum acceptable value on the local standard and  $VUF^j$  is computed as:

$$VUF^j = \frac{|V_2^j|}{|V_1^j|} \cdot 100, \quad (12)$$

with  $V_1^j$  being the measured positive sequence voltage on the j-th bus and  $V_2^j$  is the decision variable that represents the negative sequence voltage on the j-th bus.

The convexity of the cost function is assured as the  $\max(a, 0)$  function is convex: it will return 0 if  $a$  is lesser than zero and will increase linearly with  $a$  if it is non-negative, therefore, the positive semidefiniteness is ensured. Furthermore, the sum of convex functions is convex, so convexity is ensured.

In order to avoid explicitly including the  $\max()$  function on the objective function, a set of  $N$  auxiliary variables  $u^j$  is defined and two sets of  $N$  constraints are included (WILLIAMS, 2013):

$$u^j \geq 0, \quad (13)$$

$$u^j \geq VUF^j - VUF_{max}, \quad (14)$$

then, the objective function is rewritten as:

$$J = \sum_j^N u^j, \quad (15)$$

which, jointly with the constraints (13) and (14) will result in the same as (11). As the objective is minimized,  $u^j$  will assume 0 if  $VUF^j$  is lesser than  $VUF_{max}$  and  $VUF^j - VUF_{max}$  otherwise.

Ultimately, the optimization problem solved by the VUC becomes:

$$\begin{aligned} &\text{minimize } \sum_j^N u^j \\ &\text{subject to:} \end{aligned} \quad (16)$$

$$\begin{aligned} (V_2^{jDG} - V_2^j)Y_2^{jDG} &= I_2^j + \sum^M (V_2^j - V_2^m)Y_2^{jm}, \\ |I_2^{jDG}| &\leq |I_{2max}^m| \\ u^j &\geq 0 \\ u^j &\geq VUF^j - VUF_{max}, \\ &\forall j \in N, \forall m \in M \end{aligned}$$

Encompassing the model and the current limits on the constraints and the goal to keep the unbalance within acceptable limits in the objective function.

It is necessary to note that the voltages, currents, and admittances on the formulation are complex-valued numbers. In order to address complex variables in the optimization problem, they must be

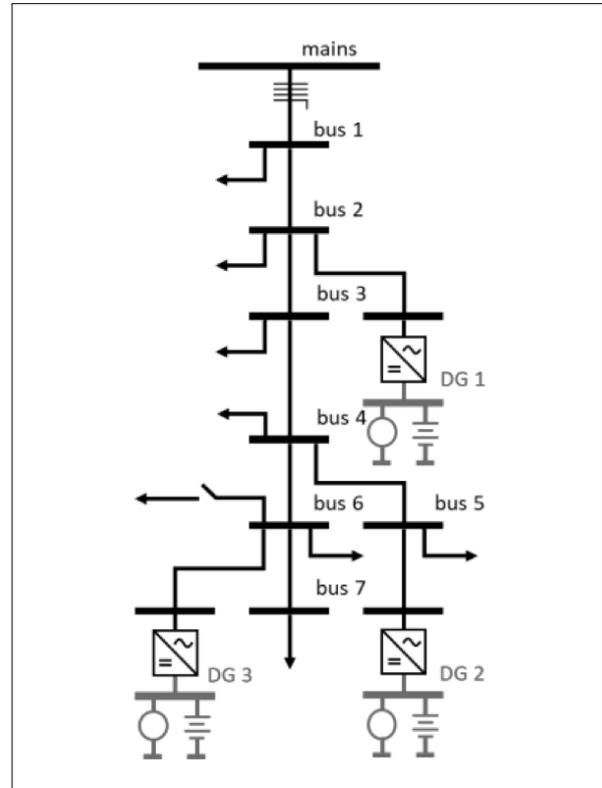
represented in terms of two other variables, one for the real part and another for the imaginary part. That said, the operations between these variables such as summation, multiplication and absolute value have to be treated accordingly. Particularly, it is noteworthy that the absolute value operations originate quadratic terms that characterize the problem as a QCQP problem. Global optimality of the solution is guaranteed, as the problem formulation is convex.

## 4 Study Case MG and VUC

As the existence of a global optimal solution for the problem of sharing the voltage unbalance compensation effort among multiple converters has been shown in the previous section by leveraging a convex formulation, the usage of the technique will be illustrated by means of a simulation.

The study case microgrid used to evaluate the proposed VUC is a seven buses low-voltage hybrid microgrid, whose AC side use a three-phase four wires system. The MG includes three distributed generators. The layout of the microgrid was adapted from Papathanassiou, Hatziargyriou and Strunz (2005) and Meng et al. (2017) and is presented in Figure 5.

Figure 5 – A 7 buses microgrid.



Source: author.

The buses 2, 5 and 6 have DGs connected to them. The converters on each of these buses are able to control the negative sequence voltage in their local buses and, therefore, are able to be employed on voltage unbalance correction.

The loads on each bus are connected in star connection and have their admittances listed in Table 1:

**Table 1** – Load buses. Admittances in siemens.

Bus	Phase A	Phase B	Phase C
$Y_1$	0.17	0.17	0.17
$Y_2$	0.07	0.07	0.07
$Y_3$	0.1-0.3j	0	0
$Y_4$	0.374	0.17	0.17
$Y_5$	0.51	0.17	0.17
$Y_6$	0.08	0.08	0.08
$Y_7$	0.1	0.1	0.1

Source: author.

The admittances of the lines are listed in Table 2. The VUC is supposed to operate on a minute sample time. On each sample, the VUC solves the optimization problem (16) and sends phase voltage set-points to each converter. An in-depth discussion of the local controllers needed in each converter is presented in Meng et al. (2015) and Meng et al. (2017). The electrical dynamics of the MG are much faster, so it is possible to assume that the MG reaches steady-state in between two samples of the VUC.

**Table 2** – Line admittances in siemens.

Bus	Phase A	Phase B	Phase C	Neutral
$Y_{01}$	0.6064 + 3.4296j	0.6064 + 3.4296j	0.6064 + 3.4296j	0.3032 + 1.7148j
$Y_{12}$	0.2538 + 1.2758j	0.2538 + 1.2758j	0.2538 + 1.2758j	0.1269 + 0.6379j
$Y_{23}$	0.6064 + 3.4296j	0.6064 + 3.4296j	0.6064 + 3.4296j	0.3032 + 1.7148j
$Y_{34}$	0.9366 + 7.846j	0.9366 + 7.846j	0.9366 + 7.846j	0.4683 + 3.923j
$Y_{45}$	0.6064 + 3.4296j	0.6064 + 3.4296j	0.6064 + 3.4296j	0.3032 + 1.7148j
$Y_{46}$	0.6064 + 3.4296j	0.6064 + 3.4296j	0.6064 + 3.4296j	0.3032 + 1.7148j

$Y_{67}$	0.6064 + 3.4296j	0.6064 + 3.4296j	0.6064 + 3.4296j	0.3032 + 1.7148j
$Y_2^{DG}$	0.6064 + 3.4296j	0.6064 + 3.4296j	0.6064 + 3.4296j	0.3032 + 1.7148j
$Y_5^{DG}$	0.6064 + 3.4296j	0.6064 + 3.4296j	0.6064 + 3.4296j	0.3032 + 1.7148j
$Y_6^{DG}$	0.6064 + 3.4296j	0.6064 + 3.4296j	0.6064 + 3.4296j	0.3032 + 1.7148j

Source: author.

## 5 Simulation Results

The study case microgrid was simulated on a 30 minutes scenario. The initial loads are described in Table 1, but the load on some of the buses are increased or decreased during the course of the simulation. The simulation was performed on MATLAB R2014a, using the solver Gurobi and the package Yalmip (LÖFBERG, 2004). It was run on an Intel Core i5-4590 processor.

At 5 minutes the admittance of the load on phase A at bus 1 is increased to 0.3101 S. At 10 minutes the admittances of the loads on phases B and C at bus 4 are decreased to 0.153 S. At 15 minutes the admittance of the load on phase A at bus 7 is increased to 0.26-0.04j S. At 20 minutes the admittance of the load on phase A at bus 6 is increased to 0.1459 S. The interval of 5 minutes between load changes was adopted for illustrative purposes, aiming to show clearly the behavior of the proposed VUC. Nevertheless, the technical performance is not hindered even if there are load changes on each sample instant.

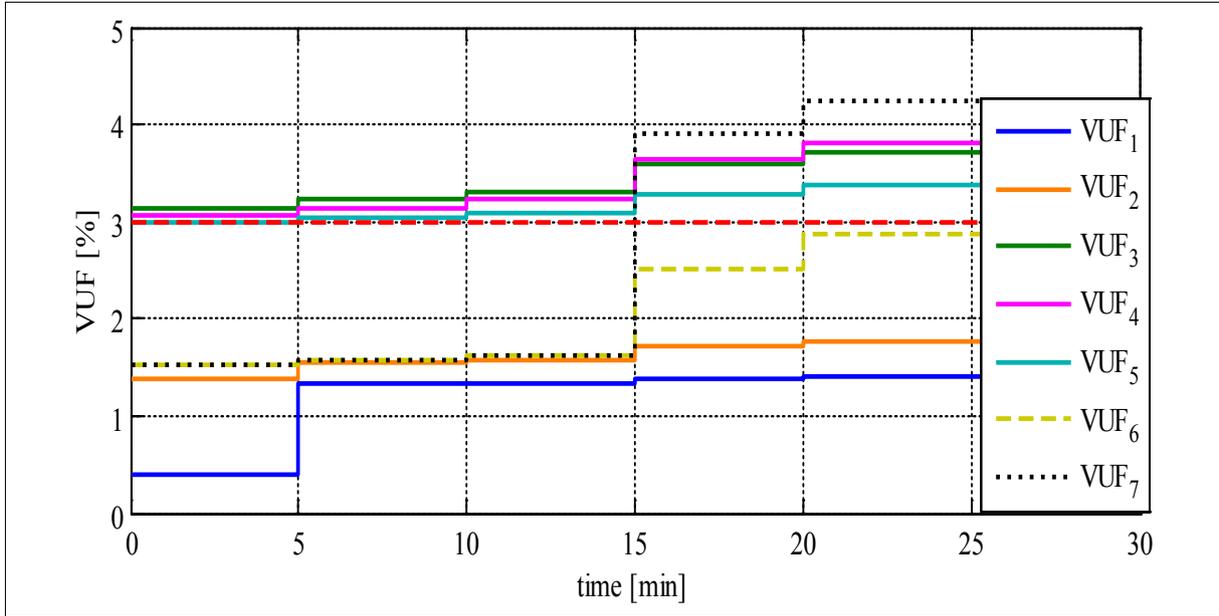
The results of the simulation, without the VUC are shown in Figure 6 (next page), that displays the evolution of the VUF on each bar as the loads change.

Without the VUC to correct the unbalance, the condition of the buses on the MG worsened at each load change. Buses 3, 4 and 5 which were already over the 3% threshold at the start of the case kept going further beyond the limit. Bus 7 which started within the acceptable limits was driven above the threshold at 15 minutes when its phase A load is increased.

In Figure 7 (next page) the results of the simulation with the VUC are shown, presenting the VUF profile on each bus as the loads change.

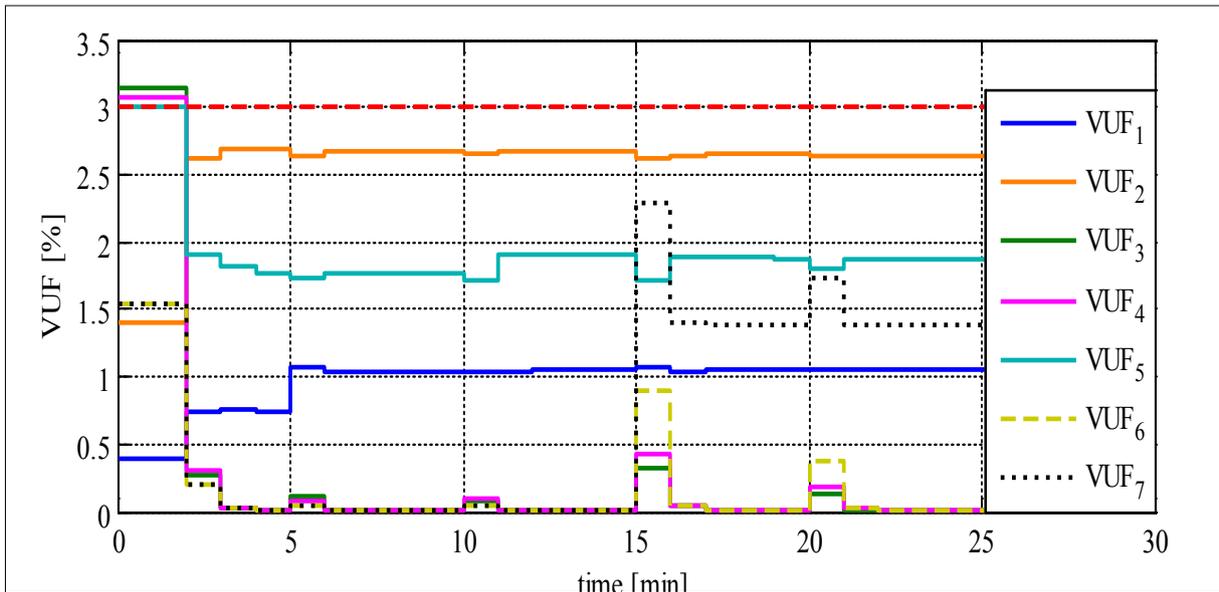
The VUC was turned on at 1 minute and, after that, it sent setpoints to the converters and lowered the VUF on each bus of the MG, making all of them comply to the 3% limit. Even with the load changes, the VUC was able to keep the unbalance under the desired level. The phase voltages outputted by each converter according

Figure 6 – Voltage unbalance profile without the VUC.



Source: author.

Figure 7 – Voltage unbalance profile with the VUC.



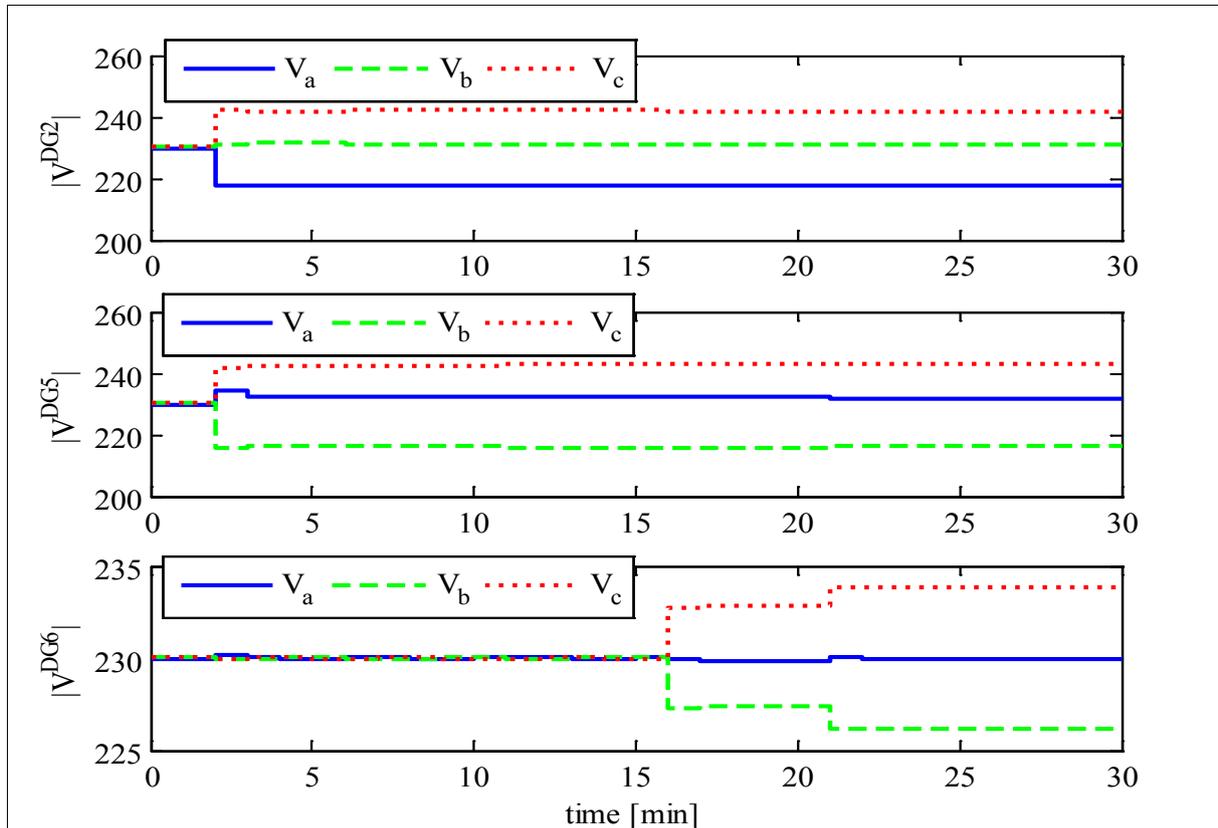
Source: author.

to the setpoint calculated by the VUC are shown in Figure 8 (next page), in RMS values.

It is intuitive that if every bus has a DG then the effort sharing problem becomes trivial and each DG will solve the unbalance of its own bus. However, on a sub-actuated condition where there are more buses than converters, the position of the DGs within the MG affects their capability of tackling the overall voltage unbalance. On the study case, the DG on bus 2 is

employed to help lowering the unbalance on buses 3 and 4, despite bus 2 not being unbalanced itself. As result of this compromise, the unbalance on bus 2 actually worsens after the VUC is turned on, although it is kept within the 3% limit. This also causes the DG on bus 7, located more distantly from buses 3 and 4 than the other two DGs, to not contribute significantly until the load changes on buses 7 and 6.

Figure 8 – Phase voltage of each converter under VUC.



Source: author.

It is also noteworthy that the running time of the VUC was 17 ms, on average, far less than the 1-minute sample period. This would enable it to operate on a faster sampling rate than the one adopted on the present work.

The proposed approach enables the overall voltage unbalance to remain under prescribed levels. Unfortunately, a thorough financial analysis of the benefits would be difficult to perform as it would encompass the degradation rate of the individual devices supplied by the MG eventually subjected to voltage unbalance. Nevertheless, the mere conformity to the energy quality standards should be regarded as a significative improvement.

Furthermore, while other results on the examined literature also point out techniques able to drive the system to the compliance of voltage unbalance standards, this work delivers an indication of the existence of global optimality. On account of this, the proposed formulation produces an optimization problem that can be addressed by any QCQP solver, obtaining a set of output voltages for the converters that minimize as much as possible the sum the voltage unbalance excesses.

## 6 Conclusions

The present work proposed a convex formulation for the voltage unbalance compensation effort sharing among distributed generators in hybrid microgrids. The VUC is based on the solution of a QCQP problem, which convexity ensures the global optimality of the solution. The formulation is based on the nodal analysis of the negative sequence equivalent circuit of the microgrid and aims to lower the overall unbalance of the network while adopting constraints to keep the VUF values bellow the given standards.

The technique was evaluated *in silico* using a study case hybrid microgrid, where the voltage unbalance would be beyond the acceptable levels unless the VUC acted on it. It was verified the capabilities of the VUC on reducing the voltage unbalance to acceptable levels.

Although the study case adopted the Brazilian standard for voltage unbalance, adapting the approach to another standard it is just a matter of choosing the appropriate  $VUF_{max}$  value on the cost function.

On future works, it is necessary to evaluate the integration of the VUC to the overall functionalities of a MGCC.

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